Number System

1. If $x = \frac{2-\sqrt{5}}{2+\sqrt{5}}$ and $y = \frac{2+\sqrt{5}}{2-\sqrt{5}}$, find the value of $x^2 - y^2$.

2. Determine rational numbers p and q if

$$\frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} = p - 7\sqrt{5}\,q.$$

3. Show that: $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2} = 5$

4. If: $x = \frac{\sqrt{p+q}+\sqrt{p-q}}{\sqrt{p+q}-\sqrt{p-q}}$, then find the value of $qx^2 - 2px + q$.

5. Show that: $\frac{x^{-1}+y^{-1}}{x^{-1}} + \frac{x^{-1}-y^{-1}}{x^{-1}} = \frac{x^2+y^2}{xy}$

6. If $2^a = 3^b = 6^c$ then show that $c = \frac{ab}{a+b}$.

7. If $x = 2 + 3\sqrt{2}$, then find the value of $\left(x + \frac{14}{x}\right)$.

8. Find the 7 rational numbers between -7/ 8 and -3 /11

9. If x= $3+2\sqrt{2}$ find $x^4 + \frac{1}{x^4}$

10. Give two rational numbers lying between 0.232332333233332...and 0.212112111211112....

11. Rationalize the denominator of the following:

(i) $\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$

(ii) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

12. If $x=\frac{1}{2-\sqrt{3}}$, prove that $x^3 - 2x^2 - 7x + 5 = 3$

13. Write a rational and irrational number between $\sqrt{2}$ and $\sqrt{2.1}$

14. Rationalize : $\frac{y^2}{y^2+x^2+x}$

15. If $x = 9 + 4\sqrt{5}$ then find the value of $\sqrt{x} - \frac{1}{\sqrt{x}}$

Polynomials

1. If a+b+c =0,find the value of $\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}$?
2. Find the remainder when $f(x) = 12x^3 - 13x^2 - 5x + 7$ is divided by $3x + 2$?
3. Find the value of a for which $(x - a)$ is a factor of the polynomial $f(x) = x^5 - a^2x^3 + 2a - 3$.
4. What must be subtracted from $4x^4 - 2x^3 - 6x^2 + x - 5$ so that the result is exactly divisible by $2x^2 + 3x - 2$?
5. When $f(x) = x^4 - 2x^3 - 3x^2 - ax + b$ is divided by $x + 1$ and $x - 1$,we get remainders 19 and 5 respectively. Find the remainder if $f(x)$ is divided by $x - 3$.
6. If $x + \sqrt{a}$ is the factor of $5x^4 - 5\sqrt{a}\,x^3 + 2x^2 - 3a + 5$, find the value of a^2.
7. Factorise :
   a) $12(x^2 + 7x)^2 - 8(x^2 + 7x)(2x - 1) - 15(2x - 1)^2$
   b) $(x^2 - 2x)^2 - 11(x^2 - 2x) + 24$
8. Find other factors of $p(x) = 4x^3 + 20x^2 + 33x + 18$, if $2x + 3$ is one of its factor?
9. If $81x^2 - y = (9x - \frac{1}{4})(9x + \frac{1}{4})$ then the value of y is ?
10. If $x^{99} + 2x^{98} + k$ is divisible by $x + 1$ then the value of k is ?
11. Factorise : $x^4 - x^3 - x^2 - x - 2$.
12. Factorise : $a^2 + b - ab - a$.
13. If $x^2 + \frac{1}{x^2} = 65$, find the value of $(x - \frac{1}{x})^3$
14. Show that $x^2 + 6x + 11$ has no zeroes.
15. Without actual division, prove that $2x^4 - 8x^3 + 3x^2 + 12x - 9$ is exactly divisible by $x^2 - 4x + 3$.